

# Short Papers

## Theory and Design of Broad-Band Nongrounded Matched Loads for Planar Circuits

L. J. PETER LINNÉR, MEMBER, IEEE, AND HANS B. LUNDÉN

**Abstract**—A filter theory approach is used for the synthesis of very broad-band resistive terminations with no direct grounding. The immediate use is in microwave integrated circuits (MIC's) where no holes or wraparound straps are desirable. Bandwidths achieved amount to 100 and 160 percent for networks of order 2 and 6, respectively. The total circuit lengths are  $\lambda$  and  $3\lambda/2$  in these cases. This performance is superior to older designs where only 30–40 percent is obtained.

### I. INTRODUCTION

Microwave systems include many devices that need resistor terminations to function properly. Among such devices, directional couplers, isolators, fixed attenuators, and some power dividers and switches may be mentioned. Common to all these devices is the need for resistive loads with a ground connection. Ground connections can be achieved in two ways, by direct connection or by virtual grounding.

The first method uses, for example, coaxial loads placed outside the circuit board and pill-box terminations on the circuit board. These solutions result in very broad-band and potential high-power terminations, but also in excessive cost and complexity. Resistive terminations can be integrated on the circuit board using resistive film, thick-film, or thin-film resistors [1], [2]. Grounding is commonly achieved by a wraparound strap at the edge of the substrate [3] or through a hole in the substrate. These designs have much lower power-handling capability. At least at higher frequencies ( $>10$  GHz), the RF ground degenerates, causing unacceptable low return loss. Additionally, edge and hole metallization as well as the drilling of holes in hard substrates are processes one seeks to avoid.<sup>1</sup>

The second method, here named virtual grounding, avoids a direct ground connection by capacitive coupling to the ground through the substrate. This is commonly done using an open-circuit stub. At its first resonance it acts as a short circuit. Virtual grounding is most suitable for microwave integrated circuits. Unfortunately, only narrow bandwidths can be designed (30 percent at return loss of 20 dB) even if low characteristic impedance stubs are used. The use of a tapered stub (for example, sectorial part of a circle) may increase this value slightly [1].

It is the purpose of this paper to use established filter theory for the design of very broad-band, virtually grounded, resistive terminations.

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L. J. P. Linnér is with the Division of Network Theory, Chalmers University of Technology, S-412 96 Gothenburg, Sweden.

H. B. Lundén was with the Division of Network Theory, Chalmers University of Technology, Gothenburg, Sweden. He is now with the Microwave Institute Foundation, S-100 44 Stockholm, Sweden.

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<sup>1</sup>The difficulties encountered in microwave integrated circuits (MIC's) apply to the corresponding monolithic circuits (MMIC's) as well. Grounding in this case is achieved by via holes [12] (pointwise elevation of the ground plane) or by connection to large grounding pads. The first method is independent of position, but is inherently inductive. The second method can only be used in border areas. It is space demanding, and grounding could be rather narrow-band in addition.

## II. THEORY OF BROAD-BAND TERMINATIONS WITH VIRTUAL GROUNDING

This section outlines the general theory of virtually grounded broad-band loads.

In [4], a method is described for the design of broad-band loads consisting of only one or two cascaded unit elements terminated in a resistance and an open-circuit stub. The design is based on the dual of a certain matching network used for circulators [5]. This design, however, is limited to networks of order  $n = 2, 3$ . Below is given a theoretical filter design for an arbitrary order.

### A. General Discussion of Virtual Grounding

In Fig. 1, a general network is shown for a virtually grounded termination.

So far, broad-banding has been achieved entirely by concentrating on the design of the network  $N_2$ .  $N_2$  is commonly an open-circuit stub with low characteristic impedance. Sometimes the stub has been linearly tapered to get increased bandwidth. The  $N_1$  network has earlier been nonexistent in matched terminations.

Fig. 2 shows the input impedance behavior of an arbitrary reactive network such as  $N_2$  in Fig. 1.

It can be shown that in any reactive network, poles and zeros intersect on the  $j\omega$ -axis. Thus, there are two ways that the short-circuit at the first zero position can be broad-banded. The first one is impedance scaling, which is restricted by the choice of wave impedances. The second one is by separating the neighboring poles as far as possible from the zero corresponding to the short-circuited frequency. This is restricted by the periodicity of distributed networks. In fact, it can be shown that the optimum reactive network in this respect is a single capacitive stub. Thus, one has to study a more complex circuit such as that in Fig. 1.

First, we will seek a more suitable equivalent network for  $R_L$  and  $N_2$  in Fig. 1. In the case of a capacitive stub in  $N_2$ , the circuits of Fig. 3 are equivalent.

From Figs. 3(b) and 1, with  $N_1$  being a reactive network, it is clear that we can treat our problem as a resistively terminated lossless two-port.

### B. Filter Approach

To investigate whether an approach as in Fig. 1 is feasible, let  $N_1$  be a cascade of unit elements. The entire circuit is shown in Fig. 4. We shall now apply established filter theory for the design of this network.

This is a high-pass filter in the terminology of Horton and Wenzel [6]. It has a bandpass characteristic centered at its first resonance and exhibits a single transmission zero at dc. Our purpose is now to seek broad-band circuits with element values within a certain realizability range.

From inspection of the network, the transmission properties are given by

$$S_{21}(p) \cdot S_{21}(-p) = \frac{p^2(1-p^2)^{n-1}}{Q_n(p^2)} \quad (1)$$

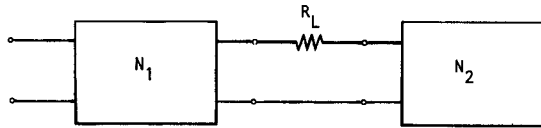


Fig. 1. A general network for virtually grounded load resistors.  $N_1$  is a matching network, and  $N_2$  is a reactive network for short-circuiting.

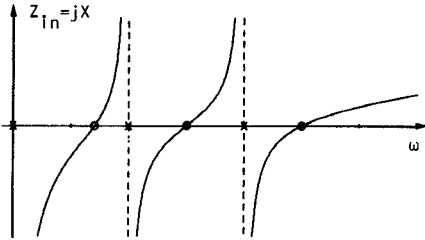


Fig. 2. Impedance of a reactive network.

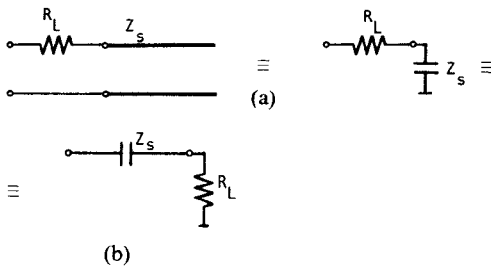


Fig. 3. (a) Virtually grounded resistor and (b) Its equivalent circuit.

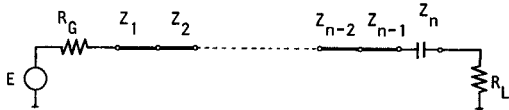


Fig. 4. Equivalent network of a broad-band, virtually grounded, resistive load connected to a generator.

where  $Q_n$  is a polynomial of order  $n$  and

$$p = \Sigma + j\Omega = \tanh(s/4f_0)$$

is the Richards variable,  $f_0$  (the center frequency) corresponds to a line length of  $\lambda/4$ , and  $s = \sigma + j\omega$  is the complex frequency variable.

It should be observed that open-circuit stubs can be inserted serially at every transmission-line junction without changing the form of (1). This is easily shown with one of the Kuroda identities. Such a network is recognized as a parallel-coupled transmission-line-resonator filter originally proposed by Cohn [7].

Different positions and numbers of open-circuit stubs can be used to scale the load resistance  $R_L$ . Carlin and Kohler [8] have shown how a wide range of transmission ratios  $R_L/R_G \leq 1$  can be designed with only a two-stub arrangement in the dual network where series-coupled open-circuit stubs are replaced by parallel-coupled short-circuit stubs. Their method is, of course, applicable to our network.

In this application, the primary goal is broad-band behavior which fortunately is in agreement with the above restriction to one single open-circuit stub in connection with the load  $R_L$ . On the contrary, the bandpass filter structure by Cohn is in practice only realizable for narrow to medium bandwidths.

An optimal filter with low losses also exhibits an optimal input reflection behavior in the frequency plane. This corresponds to

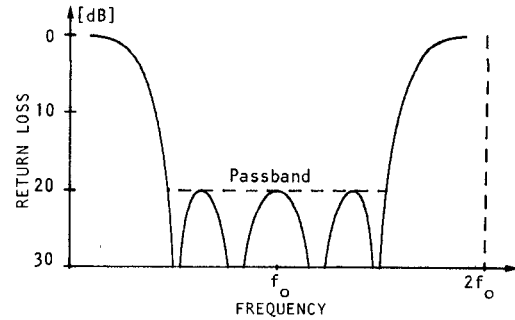


Fig. 5. Input behavior of an equiripple bandpass filter of order  $n = 4$ .

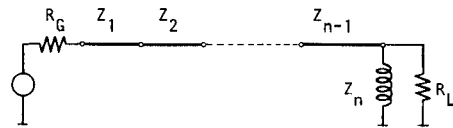


Fig. 6. The dual network of Fig. 4.

input matching. Thus, within the passband, the filter structure behaves like a matched termination. In Fig. 5, an equiripple behavior in terms of return loss is shown for a filter of order  $n = 4$ .

We may summarize that a broad-band resistive termination can be designed as a transmission-line filter with specified return loss and bandwidth.

The dual network shown in Fig. 6 can be used for similar applications, but includes a short-circuited stub. Therefore, it is less suited for our purposes. Several authors have studied this network with applications in waveguide bandpass filters and impedance transforming networks [8]–[10].

Because the dual network has exactly the same transmission properties, we may follow Riblet [9] in the design of our circuit. The characteristic function  $F$  is connected with the transfer function by

$$|S_{21}|^2 = \frac{1}{1 + \epsilon^2 F^2(x)} \quad (2)$$

In the case of an equiripple characteristic,  $F(x)$  is given by

$$\sqrt{1-x^2} F(x) = \frac{1 + \sqrt{1-x_c^2}}{2} T_n(x/x_c) - \frac{1 - \sqrt{1-x_c^2}}{2} T_{n-2}(x/x_c) \quad (3)$$

with  $T_n$  being a Chebyshev polynomial of order  $n$ ,  $x = -\cos \theta$ , and

$$\theta = \frac{\pi f}{2 f_0}$$

where  $f_0$  is the center frequency.  $x$  is related to the Richards complex frequency  $p$  by

$$x^2 = 1/(1-p^2). \quad (4)$$

If the passband is specified by

$$f_l \leq f \leq f_u \quad (5)$$

the center frequency is chosen as

$$f_0 = (f_l + f_u)/2 \quad (6)$$

and the relative bandwidth  $b$  is given by

$$b = 2 \frac{f_u - f_l}{f_u + f_l} = 2 \frac{f_u}{f_0} - 1.$$

Further, one obtains a relation for  $x_c$  above:

$$x_c = -\cos \frac{\pi f_u}{2 f_0} = \cos \frac{\pi f_l}{2 f_0}.$$

The level of the return loss  $R$  in decibels is related to  $\epsilon$  above by

$$\epsilon = \frac{1}{\sqrt{10^{R/10} - 1}}. \quad (7)$$

The synthesis of the circuit in Fig. 4 is performed by Darlington's method, resulting in network functions for a reactive two-port. Element values are finally determined from the input immittance. A detailed account of the procedure is omitted in favor of a brief description in steps. Details can be found in any standard text book.

- 1) Specify return loss  $R$ , upper/lower band limits  $f_u/f_l$ , and filter order.
- 2) From these, determine the characteristic function  $F(x)$  in (2).
- 3) Compute poles and zeros of  $F(p)$ .
- 4) Compute the poles of  $S_{21}(p)$ .
- 5) Use [11, pp. 77-84] to determine the complete two-port network matrix  $y$ .
- 6) Synthesize  $y_{11}$  according to the structure of Fig. 4.
- 7) Determine the load resistance  $R_L$  from a limiting procedure applied to  $y_{22}$  with  $p \rightarrow 0$ .

The above procedure was automated in a computer program, VIRGO. By repeated use of it, one can easily iterate to any realizable element values of  $R_L$  and  $Z_n$ .<sup>2</sup> The methods in [8] can, of course, be reformulated to arrive at expressions for  $R_L$  and  $Z_n$  before any synthesis is performed.

By formulating a synthesis for the reactive two-port, one may gain numerical advantages. The computer program VIRGO can be designed to work with polynomials of about half-order and, simultaneously, only real roots need to be handled (with few exceptions).

### C. Resulting Networks

The networks resulting from the design show some interesting properties.

1) As there normally is no specified performance outside the passband, the degree of the network can be chosen conveniently. For given bandwidth and return loss, the network degree may have to be increased to arrive at realizable element values.

2) The load resistance value used is always larger than the designed matching impedance level. This is because there is a transmission zero at dc. It is also obvious from the use of Kuroda transformations for series-connected open-circuit stub. Thus, these transformations also include a transformer that changes the impedance level of the network [10].

3) The network elements that cause a limited bandwidth range are the open-circuit stub and the unit element next to the resistor. For small bandwidths, the latter one tends towards the upper limit of the realizable wave impedance. For wide bandwidths, the open-circuit stub tends towards the corresponding lower limit [4].

<sup>2</sup>It should be noted that the values of  $R_L$  and  $Z_n$  are dependent on the design parameters such as bandwidth, network order, and return loss level. In general,  $R_L \neq R_G$ .

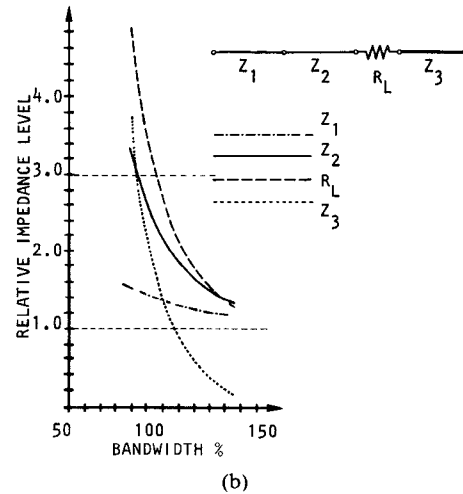
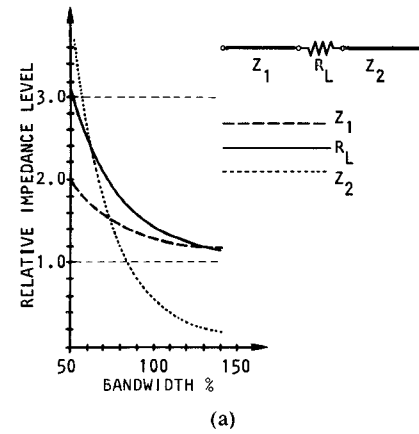


Fig. 7. Element values versus relative bandwidth for broad-band loads of order (a)  $n = 2$ , (b)  $n = 3$ . Return loss = 20 dB.

TABLE I  
SOME DESIGN LIMITS AND ELEMENT VARIATIONS FOR  
BROAD-BAND TERMINATIONS

Filter order	$Z_n = 0.5$			$Z_{n-1} = 3$		
	$W_u\%$	$f_u/f_l$	$R_L$	$W_l\%$	$f_u/f_l$	$R_L$
2	105	3.2	1.38	52	1.7	3.01
3	130	4.7	1.74	105	3.2	2.93
4	145	6.3	2.02	127	4.5	3.60
5	154	7.7	2.31	145	6.3	3.30
6	160	9.0	2.62	157	8.3	3.10
Return loss = 20 dB						

These properties are illustrated in Fig. 7 and Table I below.

The lower limit is defined by  $Z_n = 0.5$  for the open-circuit stub, and the upper limit is defined by  $Z_{n-1} = 3.0$  for the unit element next to the terminal resistor.

Empirically, the following relation is valid for the bandwidth definition  $f_u/f_l$  (upper and lower frequency limits):

$$f_u/f_l \approx 1.5n \quad (7)$$

where  $n$  is the network order.

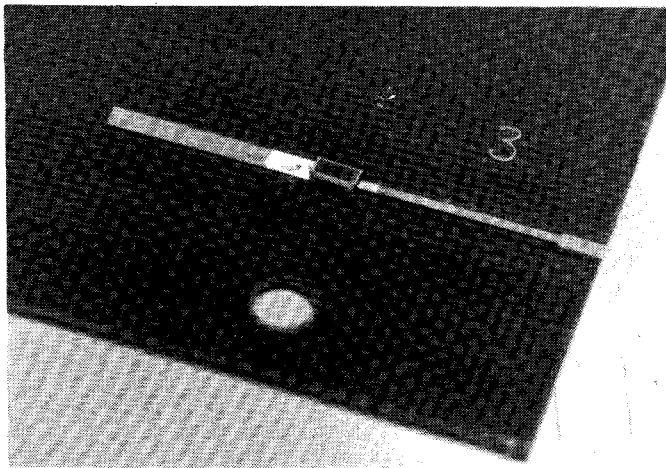


Fig. 8. Experimental second-order broad-band termination.

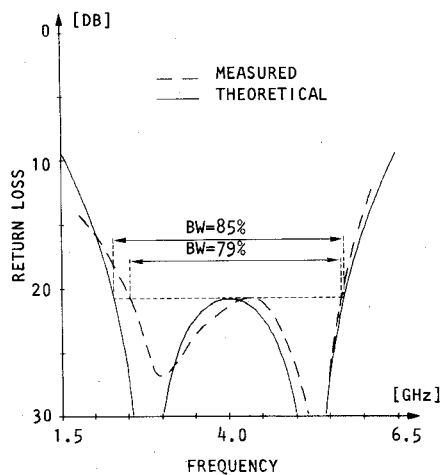


Fig. 9. Measured and theoretical responses of a second-order broad-band termination.

### III. MODIFICATIONS AND OTHER APPLICATIONS

The proposed circuit can be varied in many ways. To allow for a wider bandwidth, it is seen from Fig. 7 that the stub wave impedance must be lowered. The realizability conditions can be circumvented by using double or even triple paralleled stubs. For a higher power capability, the number of resistors can be increased by paralleling two or more resistor/stub combinations.

As mentioned briefly above, the matched load has a variety of uses wherever a grounded resistor is useful. Here only a few shall be outlined. Broad-band resistive attenuators can now be designed without any physical grounding. For optimal non-equal-ripple performance, circuit element values may have to be computer optimized through, for example, Compact®. For example, one may be interested in a specific terminal load that makes the connected  $n$ -port optimum. Such networks include isolators designed as a circulator with a terminal load at its third port, absorption loads for hybrid networks, harmonic absorbers for oscillators, etc.

### IV. MEASUREMENTS

The proposed circuit can be measured only as a one-port because the resistive load is connected symmetrically. To verify the theory, measurements have been made on a second-order circuit with a specified return loss level of 20.8 dB corresponding

TABLE II  
COMPARISON OF CANONIC AND REDUNDANT STRUCTURES  
FOR BROAD-BAND TERMINATIONS

Network	$n$	$Z_1$	$Z_2$	$Z_3$	$R_L$	Bandwidth %
Canonic	2	69.3	41.7*	-	80.0	85
Redundant	2	69.3	121.7	63.4*	185.1	85
Canonic	3	65.7	105.3	63.4*	128.3	108

to a VSWR of 1.2. The design bandwidth was 85 percent  $\Delta f_u/f_l = 2.48$ . For a 50  $\Omega$  input match, the element values were computed as  $Z_1 = 69.3 \Omega$ ,  $R_L = 80.0 \Omega$ , and  $Z_2 = 41.7 \Omega$ . Center frequency was 4.0 GHz, and stripline was used. A photograph of the layout is shown in Fig. 8.

The measured input response overlayed on the theoretical diagram is shown in Fig. 9.

The agreement is surprisingly good. No parasitics or element dissipation have been taken into account, except for the capacitance at the end of the stub. This was done by foreshortening of the stub at the center frequency.

### V. SENSITIVITY PROPERTIES

As the circuit is canonic (minimal number of elements), one can assume a certain sensitivity to errors in the circuit layout, the substrate thickness, and the dielectric constant. This has been verified from measurements. Thus, close control must be exercised on these parameters as well as on the etching procedures. The sensitivity properties can be exposed at the center frequency where the circuit behaves as a  $\lambda/4$ -transformer, relating input and load impedances as

$$Z_{in} = \frac{Z_1^2}{Z_2^2} \cdot \frac{Z_3^2}{Z_4^2} \cdots \frac{Z_{n-1}^2}{R_L} \quad (n \text{ even}) \quad (8a)$$

$$Z_{in} = \frac{Z_1^2}{Z_2^2} \cdot \frac{Z_3^2}{Z_4^2} \cdots \frac{Z_{n-2}^2}{Z_{n-1}^2} \cdot R_L \quad (n \text{ odd}). \quad (8b)$$

From (8), it is clear that errors in wave impedance are multiplied by a factor of 2. On the other hand, *systematic* errors cancel, but only for odd-order networks.

In some applications, redundant unit elements can be used to decrease the element sensitivity. From Fig. 3, it is obvious that we can add such elements from the load side with the wave impedance equal to  $R_L$ . Kuroda transformations then lead to the original structure with a number of added unit elements. If we apply this procedure to the network described in the preceding section, we arrive at the new element values shown in Table II. For comparison, it also shows a third-order network with a stub impedance equal to the second-order canonic case.

Table II also shows that this technique increases the stub wave impedance and therefore extends the range of realizable bandwidths. Another application as seen from Table II is to scale the load resistance.

### VI. CONCLUSIONS

A new theory for a very broad-band, virtually grounded, matched resistive load has been developed. The realizable bandwidth has now been extended to several octaves for microwave integrated circuits demanding no holes or wraparound straps for grounding. The technique is also applicable to resistive attenuators, integrated isolators, etc.

## ACKNOWLEDGMENT

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## A Wide-Band 12-GHz 12-Way Planar Power Divider/Combiner

VICTOR FOUAD HANNA AND JEAN JUMEAU

**Abstract**—A 12-way, low-loss, wide-band planar electrically symmetric hybrid power divider/combiner for the X-band is described. It is a two-stage fork, 12-way hybrid realized completely in microstrip. A circuit design is given to maximize the match and isolation at band center. Over a frequency band of 10–13 GHz, this divider/combiner has an insertion loss of less than 1 dB and an isolation between output ports of better than 17 dB.

## I. INTRODUCTION

Symmetric  $n$ -way power dividers/combiners have the advantage of not having either amplitude or phase power-division imbalance at all frequencies. Thus, they are used in many broadband applications such as in the feed system of multi-element antennas and as combiners of solid-state amplifiers and oscillators.

Most of the dividers/combiners described in the literature [1]–[4] are either generalizations or variations of the Wilkinson [1]  $n$ -way divider/combiner. None of them can be realized with all interconnections in the circuit plane for  $n > 2$  because they require either a resistive star network or a star of transmission lines using multilayer construction. Consequently, planar dividers/combiners might be realized using corporate structures of two-way Wilkinson split-tee [4] and hybrid circuits. The disadvantage of this approach is that the maximum value of  $n$  is

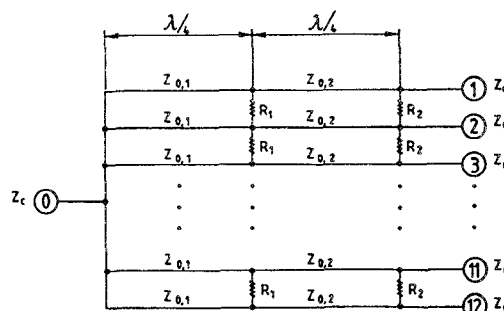


Fig. 1. A schematic representation of the two-stage fork, 12-way planar divider/combiner.

limited by the physical size and high loss of the corporate structure. The divider/combiner, which is realizable in a planar structure, was first reported by Galani and Temple [5] and it was a single-stage fork, four- or seven-way hybrid. Explicit formulas were developed by Saleh [6] for the scattering parameters of single- and two-stage fork,  $n$ -way hybrids. Saleh's [6] results show that the two-stage case gives considerably better match and isolation, and less dissipation requirements for the isolation resistors than the corresponding single-stage case, but these interesting results were not confirmed experimentally.

This paper describes the circuit design and the performance data of a 12-way planar hybrid power divider/combiner. This hybrid, which resembles the Wilkinson hybrid, is named the fork hybrid because of its geometry. It consists of two stages, each of 12-way, realized completely in a microstrip technology.

## II. CIRCUIT DESCRIPTION

A schematic diagram of the planar divider/combiner is shown in Fig. 1, where  $Z_c$  is the characteristic impedance of the input line. The divided ports are designated by the numbers 1 through 12 and are each terminated in  $Z_d$ . The characteristic impedance of each of the quarter-wave lines is  $Z_0$  and the resistance of an isolation resistor is  $R$ . The subscripts 1 and 2 refer to the first and second stages, respectively. Optimum values of circuit elements are calculated on the bases of a perfectly matched port 0 at the center frequency, a maximally flat input-output frequency response [2], [3], and a maximum of both of the match and isolation of the divided ports [6] at band center. For  $n=12$ , and for a simple match of the divided ports to 50- $\Omega$  coaxial lines,  $Z_d$  is taken to be 50  $\Omega$ , and the following optimal results are obtained:  $R_1 = 50$   $\Omega$ ,  $R_2 = 166$   $\Omega$ ,  $Z_{0,1} = 131.5$   $\Omega$ ,  $Z_{0,2} = 69$   $\Omega$ , and  $Z_c = 15.1$   $\Omega$ . The port 0 can be matched to a 50- $\Omega$  microstrip line using two quarter-wave lines of impedances 20.5 and 36.9  $\Omega$ .

## III. EXPERIMENTAL RESULTS

The 12-way divider/combiner is realized in microstrip, employing a 0.254-mm-thick Duroid substrate ( $\epsilon_r = 2.22$ ). Chips resistors of 50 and 166  $\Omega$  are soldered according to the configuration of Fig. 1. A photograph of the realized circuit is shown in Fig. 2. The divider/combiner performance is measured in the frequency band 10–14 GHz using a semi-automatic network analyzer. The average power division coefficient of the 12 output ports is plotted versus frequency in Fig. 3. The power imbalance over the output ports is  $\pm 0.45$  dB in the frequency band 10–13 GHz and  $\pm 0.8$  dB in the frequency band 10–14 GHz. Isolation coefficients between output ports have also been measured. The

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The authors are with the Division Espace et Transmission Radioélectrique, Centre National d'Etudes des Télécommunications, 38-40 Rue du Général Leclerc, 92131 Issy les Moulineaux, France.

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